

Grade 10 : IGCSE

Vectors

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Topic: Vectors Date : 17/03/2020

## Addition and Subtraction of Vectors.

### What you will learn:

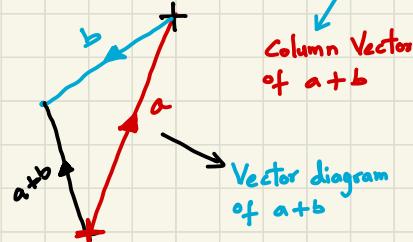
⇒ You will learn to ADD & SUBTRACT vectors and represent them using diagram

### Examples

$$\text{If } \mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

### 1) Find $\mathbf{a} + \mathbf{b}$

Solution  $2 + -3 = 2 - 3 = -1$   
 $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

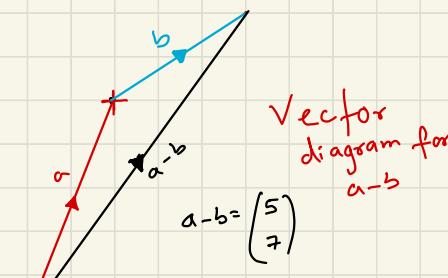


### 2) Find $\mathbf{a} - \mathbf{b}$

NOTE: we can write  $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$

$$\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

$$\mathbf{a} + (-\mathbf{b}) \rightarrow \mathbf{b} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}, -\mathbf{b} = -\begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$



$$\mathbf{a} - \mathbf{b}$$

$$\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

### Exercise 31.2

In the following questions,

$$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}, \mathbf{d} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Draw vector diagrams to represent the following:

$$\mathbf{a} \quad \mathbf{a} + \mathbf{b}$$

$$\mathbf{b} \quad \mathbf{b} + \mathbf{a}$$

$$\mathbf{c} \quad \mathbf{a} + \mathbf{d}$$

$$\mathbf{d} \quad \mathbf{d} + \mathbf{a}$$

$$\mathbf{e} \quad \mathbf{b} + \mathbf{c}$$

$$\mathbf{f} \quad \mathbf{c} + \mathbf{b}$$

What conclusions can you draw from your answers to Question 1 above?

### 3 Draw vector diagrams to represent the following:

$$\mathbf{a} \quad \mathbf{b} - \mathbf{c}$$

$$\mathbf{b} \quad \mathbf{d} - \mathbf{a}$$

$$\mathbf{c} \quad \mathbf{a} - \mathbf{c}$$

$$\mathbf{d} \quad \mathbf{a} + \mathbf{c} - \mathbf{b}$$

$$\mathbf{e} \quad \mathbf{d} - \mathbf{c} - \mathbf{b}$$

$$\mathbf{f} \quad \mathbf{c} + \mathbf{b} + \mathbf{d}$$

### 4 Represent each of the vectors in Question 3 by a single column vector

$$\mathbf{d} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}, -\mathbf{a} + (-\mathbf{c})$$

$$\mathbf{d} + (-\mathbf{c}) + (-\mathbf{b})$$

$$-\mathbf{c} = -\begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$-\mathbf{b} = -\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\mathbf{d} - \mathbf{c} - \mathbf{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

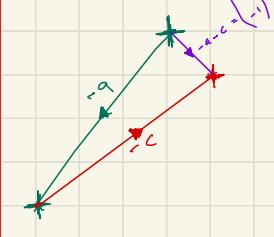
$$\mathbf{x} = 3 + -4 + -2 = 9$$

$$\mathbf{y} = -2 + -3 + -1 + +1 + -1 = 0$$

$$\mathbf{d} - \mathbf{c} - \mathbf{b}$$

$$\mathbf{d} - \mathbf{c} - \mathbf{b} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$$

$$-\mathbf{c} = -\begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, -\mathbf{a} = -\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ -7 \end{pmatrix}$$



Topic: VectorsMultiplying a vector by a scalar

Learning Objective: You will learn to multiply a vector by a scalar.

- Scalar quantity is that with only magnitude. Vector quantity has both magnitude & direction.

Example:

If  $\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$  find:

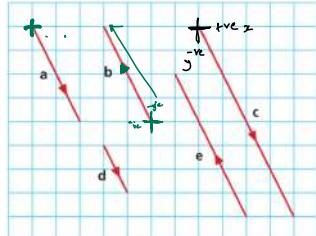
$$\textcircled{1} \quad 2\mathbf{a} = 2 \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$\textcircled{2} \quad \frac{1}{2}\mathbf{a} = \frac{1}{2} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$$

$$\textcircled{3} \quad -3\mathbf{a} = -3 \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

Example 2

If  $\mathbf{a} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ , express the vectors  $\mathbf{b}$ ,  $\mathbf{c}$ ,  $\mathbf{d}$  and  $\mathbf{e}$  in terms of  $\mathbf{a}$ .



$$\boxed{\mathbf{b} = -\mathbf{a}} \quad \boxed{\mathbf{c} = 2\mathbf{a}} \quad \boxed{\mathbf{d} = \frac{1}{2}\mathbf{a}} \quad \boxed{\mathbf{e} = -\frac{1}{2}\mathbf{a}}$$

$$\mathbf{c} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

$$\boxed{\mathbf{a} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}}$$

$$\boxed{\mathbf{C} = 2\mathbf{a}}$$

$$\mathbf{a} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$\mathbf{b}$  = get the column vector of  $\mathbf{b}$

$$\mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

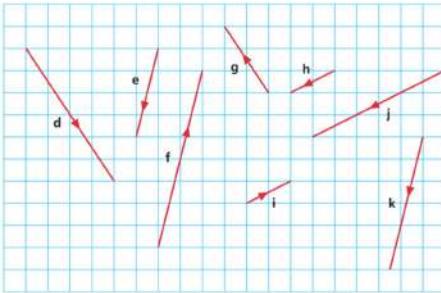
$$\mathbf{b} = -1(\mathbf{a}) = -1 \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \mathbf{b}$$

$$\mathbf{b} = -1\mathbf{a}$$

## Exercise 31.3

$$\textcircled{1} \quad \mathbf{a} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

Express the following vectors in terms of either  $\mathbf{a}$ ,  $\mathbf{b}$  or  $\mathbf{c}$ .



$$\textcircled{2} \quad \mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Represent each of the following as a single column vector:

$$\begin{array}{lllll} \textcircled{a} \quad 2\mathbf{a} & \textcircled{b} \quad 3\mathbf{b} & \textcircled{c} \quad -\mathbf{c} & \textcircled{d} \quad \mathbf{a} + \mathbf{b} & \textcircled{e} \quad \mathbf{b} - \mathbf{c} \\ \textcircled{f} \quad 3\mathbf{c} - \mathbf{a} & \textcircled{g} \quad 2\mathbf{b} - \mathbf{a} & \textcircled{h} \quad \frac{1}{2}(\mathbf{a} - \mathbf{b}) & \textcircled{i} \quad 2\mathbf{a} - 3\mathbf{c} & \end{array}$$

$$\mathbf{j} = \frac{3}{2}\mathbf{b}$$

$$\mathbf{i} = \frac{1}{2}\mathbf{b}$$

$$\mathbf{c} = -2\mathbf{a}$$

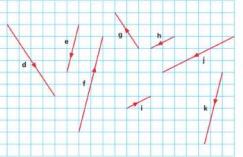
## Topic: Vectors

### Exercise 31.3 Question 2

Exercise 31.3

$a = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$	$b = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$	$c = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$
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Express the following vectors in terms of either  $a$ ,  $b$  or  $c$ .



$$2 \ a = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \ b = \begin{pmatrix} -4 \\ -1 \end{pmatrix}, \ c = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Represent each of the following as a single column vector:

- $a + 2a$   A  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$   B  $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$   C  $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$
- $f - 3c - a$   D  $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$   E  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$   F  $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$
- $3b - a$   G  $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$   H  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$   I  $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$
- $g + b - c$   J  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$   K  $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$   L  $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$$a = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \ b = \begin{pmatrix} -4 \\ -1 \end{pmatrix}, \ c = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$\textcircled{a} \ 2a, \ 2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

$$\textcircled{b} \ 3c - a, \ 3 \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -6 \\ 12 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix}$$

$$\textcircled{c} \ \frac{1}{2}(a-b), \ \frac{1}{2} \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$(i) 2a - 3c$$

$$2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} -4 \\ -1 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} -6 \\ -3 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

## Topic: Vectors

### Magnitude of a Vector

#### Points to note

⇒ Magnitude of a vector is given by its length.

⇒ The longer the length, the greater the magnitude.

⇒ The shorter the length, the smaller the magnitude.

⇒ It is calculated using pythagoras' theorem.

#### Notation:

Given a vector,  $b$  or  $\vec{bc}$

## Objective: You will learn to calculate

the magnitude of a vector.

### Exercise 31.4

Calculate the magnitude of the following vectors, giving your answers to 1.d.p.

$$a: \vec{AB} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \ b: \vec{BC} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \ c: \vec{CD} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$$

$$d: \vec{DE} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}, \ e: 2\vec{AB} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}, \ f: 2\vec{CD} = \begin{pmatrix} -8 \\ -12 \end{pmatrix}$$

$$\textcircled{3} \ a = \begin{pmatrix} 4 \\ -3 \end{pmatrix}, \ b = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \ c = \begin{pmatrix} -1 \\ -8 \end{pmatrix}$$

Calculate the magnitude of the following, giving your answers to 1.d.p.

$$\textcircled{4} \ a + b = \begin{pmatrix} -1 \\ -4 \end{pmatrix}, \ \textcircled{5} \ 2a - b = \begin{pmatrix} 6 \\ -3 \end{pmatrix}, \ \textcircled{6} \ b - c = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \ \textcircled{7} \ a + 2b - c = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\textcircled{1} \ c = \begin{pmatrix} -1 \\ -8 \end{pmatrix}, \ b = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$$

$$2c = 2 \begin{pmatrix} -1 \\ -8 \end{pmatrix} = \begin{pmatrix} -2 \\ -16 \end{pmatrix}, \ 3b = 3 \begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} -15 \\ 21 \end{pmatrix}$$

$$2c + 3b$$

$$\begin{pmatrix} -2 \\ -16 \end{pmatrix} + \begin{pmatrix} -15 \\ 21 \end{pmatrix} = \begin{pmatrix} -17 \\ 5 \end{pmatrix}$$

$$|2c + 3b| = \sqrt{(-17)^2 + 5^2} = \sqrt{289 + 25} = \sqrt{314} = 18.14 = \underline{\underline{18.14}}$$

$$\textcircled{f} \ a + 2b - c$$

$$a = \begin{pmatrix} 4 \\ -5 \end{pmatrix}, \ b = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \ c = \begin{pmatrix} -1 \\ -8 \end{pmatrix}$$

$$2b = 2 \begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} -10 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} -10 \\ 14 \end{pmatrix} - \begin{pmatrix} -1 \\ -8 \end{pmatrix} = \begin{pmatrix} -5 \\ 19 \end{pmatrix}$$

$$= \sqrt{(-5)^2 + 19^2}$$

$$= 25 + 361$$

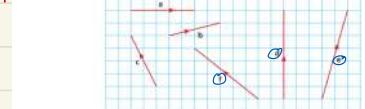
$$= 386$$

$$= 19.646$$

$$= \underline{\underline{19.6}}$$

### Exercise 31.4

Calculate the magnitude of the vectors shown below. Give your answers correct to 1.d.p.



Find the magnitude of

$$a: \vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$|a| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$b: \vec{bc} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$$

$$|\vec{bc}| = \sqrt{(-1)^2 + 8^2} = \sqrt{1 + 64} = \sqrt{65} = \underline{\underline{8.06}}$$

$$c: \vec{c} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$|\vec{c}| = \sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5} = \underline{\underline{4.47}}$$

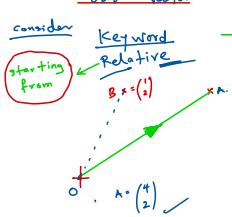
$$d: \vec{d} = \begin{pmatrix} 0 \\ 7 \end{pmatrix} = \sqrt{0^2 + 7^2} = \sqrt{49} = 7.0$$

$$e: \vec{e} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \sqrt{2^2 + 7^2} = \sqrt{24 + 49} = \sqrt{73} = 2.73 \quad (1dp)$$

$$f: \vec{f} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} = \sqrt{(-5)^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41} = \underline{\underline{6.45}}$$

## Topic: Vectors

### Position Vector

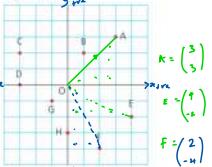


Objective: You will learn to give position

vector relative to a given point.

### Exercise 31.5

- 1 Give the position vectors of A, B, C, D, E, F, G and H relative to O in the diagram (below).



### Opposite Vectors

same length opposite direction

$$\vec{AB} = \vec{CD} = \vec{EF} = a$$

$$\vec{CD} = b$$

$$\vec{DC} = -b$$

$$\vec{AE} = \vec{GB}$$

$$\vec{EA} = -a$$

$$\vec{AE} = b$$

$$\vec{EA} = -a + b$$

$$= b - a$$

- 1 If  $\vec{AG} = a$  and  $\vec{AE} = b$ , express the following in terms of  $a$  and  $b$ :

$$\begin{aligned}\vec{GE} &= \vec{GA} + \vec{AE} + \vec{EG} \\ &= -b + a + b \\ &= a\end{aligned}$$

$$\begin{aligned}\vec{EB} &= \vec{EA} + \vec{AB} \\ &= -a + b\end{aligned}$$

$$\begin{aligned}\vec{HC} &= \vec{CA} + \vec{AH} + \vec{CH} \\ &= a + b + b \\ &= 2b\end{aligned}$$

2. If  $\vec{LM} = a$  and  $\vec{LN} = b$ , express the following in terms of  $a$  and  $b$ :

$$\begin{aligned}\vec{FD} &= \vec{FC} + \vec{CD} + \vec{DB} \\ &= b + -a + -a \\ &= b + -2a \\ &= \underline{\underline{b - 2a}}$$

$$\begin{aligned}\text{(a)} \quad \vec{IP} &= a \quad \vec{LR} = b \\ \text{(b)} \quad \vec{LM} &= \vec{LP} + \vec{PM} \\ &= a + a = 2a \\ \text{(c)} \quad \vec{PQ} &= b \\ \text{(d)} \quad \vec{PR} &= \vec{PL} + \vec{LR} \\ &= -a + b \\ \text{(e)} \quad \vec{PQ} &= b - a\end{aligned}$$

### Parallel Vectors

different length different direction  
same length same direction  
multiples of each other

$$\begin{aligned}\vec{AB} &= a \\ \vec{CD} &= 2a \\ \vec{EF} &= a \\ \vec{GH} &= -a \\ \vec{IJ} &= -1 \times \vec{AB} \\ \vec{KL} &= -1 \times \vec{AG}\end{aligned}$$

### Vector Addition

$$\begin{aligned}\vec{AC} &= \vec{AB} + \vec{BC} \\ &= a + b \\ &= a + b\end{aligned}$$

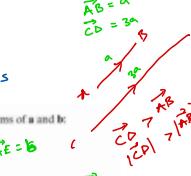
## Topic: Vectors

### Vector Geometry

Objective: You will learn to solve problems involving vector geometry.

### Exercise 31.6

magnitude  $\rightarrow$  length  
direction



#### \* Equal vectors

#### \* Opposite vectors

#### \* Vector addition

#### \* Parallel vectors

1 T is the midpoint of the line PT and H divides the line QS in the ratio 3:2.  
i) Express each of the following in terms of  $a$  and  $b$   
ii)  $TS = ?$   
iii) Show that  $HT = \frac{1}{4}(a + b)$

$$\text{(i)} \quad \vec{PS} = 2 \vec{PT} = 2a$$

$$\vec{PS} = 2 \vec{PT}$$

$$\vec{PT} = \frac{1}{2} \text{ of } \vec{PS}$$

$$\vec{PT} = \frac{1}{2}b$$

$$\therefore \vec{QS} = -b + 2a$$

$$= 2a - b \parallel$$

$$\text{(ii)} \quad \vec{PQ} = \vec{PQ} + \vec{QF}$$

$$\vec{PQ} = b$$

$$\vec{QF} = \frac{1}{4} \text{ of } \vec{QS}$$

$$= \frac{1}{4} \times 2a - b$$

$$= \frac{2a}{4} - \frac{b}{4} = \left(\frac{a}{2} - \frac{b}{4}\right)$$

$$\vec{QF} = \frac{(2a - b)}{4}$$

$$\vec{PR} = b + \frac{2a - b}{4}$$

$$= \frac{b}{4} + \frac{2a - b}{4}$$

$$= \frac{4b + 2a - b}{4}$$

$$\vec{PR} = \frac{3b + 2a}{4}$$

$$\vec{PT} = \frac{1}{4}(2a - 3b)$$

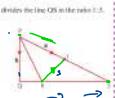
$$\vec{PT} = \vec{RP} + \vec{PT}$$

$$\vec{PT} = \frac{3b + 2a}{4}$$

$$\vec{RP} = -\frac{3b - 2a}{4}$$

$$\vec{PT} = a$$

$$\begin{aligned}\vec{PT} &= \frac{-3b - 2a}{4} + \frac{a}{1} \\ &= \frac{-3b - 2a + 4a}{4} = \frac{2a - 3b}{4} \\ &= \frac{1}{4}(2a - 3b)\end{aligned}$$



$$\vec{PT} = \vec{TS}$$

$$\vec{TS} = \frac{1}{3} \text{ of } \vec{PR}$$

$$\vec{TS} = \frac{2}{3} \vec{PR}$$

$$\therefore \vec{TS} = \frac{2}{3} \vec{PR}$$

$$\vec{PT} = \frac{1}{4}(2a - 3b)$$

$$\vec{PT} = \vec{RP} + \vec{PT}$$

$$\vec{PT} = \frac{3b + 2a}{4}$$

$$\vec{RP} = -\frac{3b - 2a}{4}$$

$$\vec{PT} = a$$