Lecture 1 – Elementary Number Theory I
Instructor: Nicholas W.

HCF (Highest Common Factor) of two or more numbers is the highest number among all the common factors of the given numbers

- HCF of 60 and 40 is 20

  i.e HCF (60, 40) = 20

Methods

- HCF by listing **factors method**
- HCF by **prime factorization**
- HCF by **division method**

Example: Listing method

Find the HCF of 30 and 42.

Solution:

List the factors of 30 and 42.

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30

factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

Clearly, 1, 2, 3, and 6 are the common factors of 30 and 42. But 6 is the greatest of all the common factors. Hence, the HCF of 30 and 42 is 6.

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Example: **Prime Factorization**

Find the HCF of 60 and 90.

Solution:

The prime factors of 60 = $2 \times 2 \times 3 \times 5$ or $2^2 \times 3 \times 5$
The prime factors of 90 = $2 \times 3 \times 3 \times 5$ or $2 \times 3^2 \times 5$.

Now, the HCF of 60 and 90 will be the product of the common prime factors that have the least powers, which are, 2, 3, and 5. So, HCF of 60 and 90 = $2 \times 3 \times 5 = 30$

Example: Division method.

Solution: Find the HCF of 198 and 360. Let us find the HCF of the given numbers using the following steps.

- **Step 1:** Among the two given numbers, 360 is the larger number, and 198 is the smaller number.
- **Step 2:** We divide 360 by 198 and check the remainder. Here, the remainder is 162. Make the remainder 162 as the new divisor and the previous divisor 198 as the new dividend and perform the long division again.
- **Step 3:** We will continue this process till we get the remainder as 0. Here, the last divisor is 18 which is the HCF of 198 and 360. Therefore, the HCF of 198 and 360 is 18.

Practice Question – Try now !
Find the HCF of 6, 72, and 120 by using the listing factors method.

The **LCM (least common multiple)** of two or more numbers is the smallest number among all common multiples of the given numbers. Let us take two numbers, 2 and 5. Each will have its own set of multiples

- Multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20,…. 2n, for $n \in Z+$
- Multiples of 5 are 5, 10, 15, 20, …., 5n , for $n \in Z+$

Thus, the common multiples of 2 and 5 are 10, 20, and so on. The smallest number among 10, 20, and so on is 10. So the least common multiple of 2 and 5 is 10. It can be written as LCM (2, 5) = 10.

Methods

- LCM by listing **factors method**
- LCM by **prime factorization**
- LCM by **division method**

Example: Factor Method. Find the least common multiple (LCM) of 4 and 5.

Solution: The first few multiples of 4 are: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, ...

And the first few multiples of 5 are: 5, 10, 15, **20**, 25, 30, 35, **40**, ...

We can observe that 20 is the least multiple which is common in the multiples of 4 and 5. Therefore, the least common multiple (LCM of 4 and 5) is 20.

Example: Prime Method - Find the least common multiple (LCM) of 60 and 90 using prime factorization.

Solution: Let us find the LCM of 60 and 90 using the prime factorization method.

- **Step 1:** The prime factorization of 60 and 90 are: 60 = 2 × 2 × 3 × 5 and 90 = 2 × 3 × 3 × 5
- **Step 2:** If we write these prime factors in their exponent form it will be expressed as, $60 = 2^2 \times 3^1 \times 5^1$ and $90 = 2^1 \times 3^2 \times 5^1$
- **Step 3**: Now, we will find the product of only those factors that have the highest powers among these. This will be, $2^2 \times 3^2 \times 5^1 = 4 \times 9 \times 5 = 180$

Therefore, LCM of 60 and 90 = 180.

Example: Division Method - Find the least common multiple (LCM) of 6 and 15 using the division method.

Solution: Let us find the least common multiple (LCM) of 6 and 15 using the division method using the steps given below.

- **Step 1:** 2 is the smallest prime number and it is a factor of 6. Write 2 on the left of the two numbers. For each number in the right column, continue finding out prime numbers which are their factors.
- **Step 2:** 2 divides 6 but it is not a factor of 15, so we write the number 15 in the row below as it is. Continue the steps until 1 is left in the last row. Then, we divide 3 and 15 by 3. This gives us 1 and 3. Now, again we write 5 on the left side and we finally get 1, 1 as the quotient in the last row.
- **Step 3:** Then we multiply these numbers on the left. The LCM is the product of all these prime numbers. LCM of 6 and 15 is, 2 × 3 × 5 = 30.

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Practice Question:
Find the LCM of 25, 15, and 30 using the prime factorization method.

Elementary Number Theory

How do you write a function representing odd and even sequence ? (Hint : discussed in lecture)
i.e Multiples of 2 = 2n $\{n \in Z\}$

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Prime number important properties of prime numbers are given below:

- A prime number is a whole number greater than 1.
- It has exactly two factors, that is, 1 and the number itself.
- There is only one even prime number, that is, 2.
- Any two prime numbers are always co-prime to each other.
- **Every number can be expressed as the product of prime numbers**

Practice Questions:

1) State true or false with respect to prime numbers.

a.) 1 is a prime number.

b.) The only even prime number is 2.

c.) The first five prime numbers are 2, 3, 5, 7, and 9.

d.) All prime numbers are odd.

2) List the first 30 Prime numbers

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Challenge corner

1) Prove that the sum of 3 consecutive integers is always divisible by 3
2) Find all $n \in Z+$ such that $n^2 + 1$ is divisble by $n + 1$